Private Peering Among Internet Backbone Providers*

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Abstract

We develop a model, in which Internet backbone providers decide on private peering agreements, comparing the benefits of private peering relative to being connected only through National Access Points. Backbone providers compete by setting capacities for their networks, capacities on the private peering links, if they choose to peer privately, and access prices. The model is formulated as a multistage game. We examine the model from two alternative modelling perspectives - a purely non-cooperative game, where we solve for Subgame Perfect Nash Equilibria through backward induction, and a network theoretic perspective, where we examine pairwise stable and efficient networks. While there are a large number of Subgame Perfect Nash Equilibria, both the pairwise stable and the efficient network are unique and the stable network is not efficient and vice versa. The stable network is the complete network, where all the backbone providers choose to peer with each other, while the efficient network is the one, where the backbone providers are connected to each other only through the National Access Points.

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1 Introduction

The Internet started largely as a public sector endeavour, but subsequently became increasingly commercialized. In 1995, the privatization of the Internet was complete when the NSFNET was replaced by National Access Points (hereafter NAPs) and four commercial backbones. Given the nature of very strong positive network externalities and the need for consumers to have access to all possible websites, sharing of network infrastructure has become something of a necessity. Two main forms of interconnection emerged - peering under which backbones carry each other's traffic without charging each other and transit under which the downstream provider pays the upstream provider a certain settlement payment for carrying its traffic.

A large part of the recent literature on the economic aspects of the Internet has been devoted to these arrangements. It is, for instance, well known that the Internet usage is subject to the problem of the commons, making peering between providers inviable (Little and Wright, 2000). Besides, when the Internet backbone providers engage in private peering they have to make large investments in the fiber optic capacity. In spite of that, large backbone providers, or so called Internet Access Providers (hereafter IAPs) do engage in private peering with one another.

In this paper we will try to examine the incentives behind the backbone providers' decisions to engage in private peering. First a four stage game is considered in this paper. In the first stage IAPs decide how they want to be connected to the other IAPs, i.e., the IAPs decide whether to connect to other IAPs through private connections and NAPs or only through NAPs. Once an IAP has made a decision on the types of connection with other IAPs, then it chooses a capacity for its network, determining how many customers it can handle at a certain point in time. The difference between the IAP's network capacity and its demand determines connection failure rates. In the third stage, if an IAP's decision was to engage in private peering, then the IAP chooses capacities for the links to connect to other IAPs, which determines usage congestion on the private link. This along with the congestion at the NAPs determines the overall congestion. In the last stage the IAPs compete a' la Bertrand. If the IAPs choose not to engage in private peering in the first stage, then they compete by determining their network capacities and prices only. We focus on a market with three IAPs only. We consider a decentralized decision making model, where the choices made by each IAP affect the outcomes and choices made by the other IAPs and equilibrium is reached without any central control.

Subsequently, we look at the model from a network perspective by redefining the first stage as a link formation stage. We examine the properties of pairwise stable and efficient networks.

Again we analyze networks with three IAPs, where the possibility of peering between two providers makes a third non-peering provider vulnerable to the loss of demand and, hence, profitability. Such a possibility is nonexistent in two-provider networks making peering unlikely. In other words, our results captures the difference of the dynamics in a three provider network relative to a two-provide network which other papers do not address. Our main results are that while there is a multitude of subgame perfect Nash equilibria resulting in a multitude of network configurations, the stable and efficient networks are unique. Furthermore, the stable network, while not efficient, is the one where all IAPs make private peering agreements with the rest of the IAPs. On the other hand the efficient network, while not stable, is the one where none of the IAPs peer privately, i.e., they exchange traffic with each other only through NAPs.

The paper is organized as follows. Section 2 examines the history of the Internet, the Internet architecture and infrastructure, and interconnection arrangements of the Internet service provision. Section 3 discusses economic literature on the Internet service provision. In Section 4 we present our model. The conclusion follows in Section 5.

2 Background

In this stage we briefly outline the evolution of the Internet and the features that make it unique relative to other markets.

2.1 Internet Architecture and Infrastructure

The Internet is a worldwide system of interconnected computer networks, in which users at any computer can (if they have permission) communicate with any other computer in the Internet (Hall, 2000, http://whatis.techtarget.com).

First we will briefly talk about the evolution of the Internet. In early 1960s, as the computers became crucial to the national defense, the U.S. Department of Defense began to search ways to share computing resources of major research centers and institutions. The purpose was to create a worldwide network that would not require a centralized control, so that the network would operate, even if some parts of it fail. On the other hand it was important to exchange resources despite having different systems, different languages, hardware and network devices.

In 1969 the Advanced Research Projects Agency (ARPA) of the Department of Defense developed ARPANET, the first wide area packet switching network, which allowed individual units of data to be transmitted from one computer to another as independent entities. Messages could be routed and rerouted in more than one directions, so the network could operate even if some parts of it fail.

At first four computers were connected through this network, University of California at Los Angeles, SRI International, the University of California at Santa Barbara, and the University of Utah. Then over the following years many researchers and academic institutions were connected to the network. The researchers at other universities were developing their own networks. The networking software has become more widely used by academic and research institutions, as the use of personal computers increased in 1980s. In late 1980s the independent networks merged into one. The Department of Defense and most of the academic networks comprising the Internet were receiving funds from National Science Foundation, which restricted commercial traffic on its networks. In 1991 restrictions on the Internet commercial traffic were lessened and by 1995 NSF completed the privatization of the Internet. After the privatization four companies: Pacific Bell, Sprint, Ameritech and MFS Corporation, became owners of four Network Access Points (NAP), located in San Francisco, New York, Chicago and Washington D.C.. The companies, so called backbone providers, exchanged traffic with each other at NAPs. The backbone providers were selling Internet access rights to other large companies, so called Internet service providers, which in turn were providing services to smaller firms and individuals (Schneider and Perry, 2001).

The networks that comprise the Internet are self deterministic and autonomous, and communicate with each other without being controlled by a central authority.

The role of each network cannot be easily predicted in advance, as the Internet is based on connectionless transmission technology. No dedicated connection is required and no dedicated route has to be set up between the sender and the receiver, because the Internet uses packet switching technology[1] to transfer data across the network. The outgoing data is converted to a format, usable by the local network medium, then data files are broken down into so called packets or datagrams, labeled with codes, which have information on their origin and destination. Each packet is transmitted over the Internet and reassembled at the destination. A datagram formatting and addressing mechanism is independent of any specific characteristics of the individual networks comprising the Internet (Hall, 2000).

The operation of the Internet is supported mainly by two basic protocols: Transmission Control Protocol (TCP) and the Internet Protocol (IP) (Schneider and Perry, 2001), a software based set of networking protocols that allow any system to connect to any other system using any network topology (Hall, 2000). IP protocol is responsible for routing individual packets from their origin to their destination. Each computer has at least one globally unique address, called its IP address, that identifies it from all other computers in the Internet. The IP address has information on both the network, the computer it belongs to, as well as its location in that network. Currently used IPv4 uses a 32 bit number for an IP address. The next generation Internet, IPv6, will use a 128 bit number for an IP address. Each packet transmitted over the Internet contains both the sender's IP address and the receiver's IP address. The datagrams are transmitted from one host to another, one network at a time (Hall, 2000). Each packet of a data file might take a different path, but it will end up at the destination ready to be reassembled. The best route for transmitting a packet from the origin to its destination is determined at each router-computer that the packet passes on its trip. The router's decision about where to send the packet depends on its current understanding of the state of the networks it is connected to. This includes information on available routes, their conditions, distance and cost. The packets, having the same origin and destination, travel across any network path that the routers or the sending system consider most suitable for that

[1] An alternative of packet switched networks is a circuit-switched networks. In circuit switched networks (like the telephone network) each connection between the sender and the reciever requires a dedicated path for the duration of the connection.

packet at each point of time. If at some point in time some parts of the network do not function, the sending system or a router between the origin and destination will detect the failure and would forward the packet via a different route (Telegeography, 2000).TCP controls the assembly of data into packets before the transmission, keeps track of the individual packets of the data and controls reassembly of the packets at the destination.

The networks in the Internet interconnect and exchange data based on several settlements (Telegeography, 2000):

- Sender Keeps All (SKA), neither network counts or charges for traffic exchange;
- Unilateral settlement or transit, the downstream customer pays the upstream provider to carry its traffic;
- Bilateral settlement, two providers agree on price, taking into account the imbalance in exchanged traffic;
- Multilateral settlement, several providers construct shared facilities and share the costs.

The type of settlement chosen depends on the Internet Service Providers' (hereafter ISPs') size, domestic and international capacity, network quality, content and customer profile, and routing and interconnection topology. (Telegeography, 2000). At the early stages of the Internet development the networks comprising the Internet were closer in size and had comparable traffic flows. So they used to exchange traffic as "peers", i.e. not paying each other for the exchange of traffic (SKA settlement). As the Internet became more commercial, the size of networks have changed. Then larger networks started to change peering agreements. Now smaller networks pay larger networks for connectivity (transit), but larger networks still exchange traffic under peering.

Cukier (1998a) proposes a functional classification of ISPs based on four classes, which shows the asymmetry in traffic interchange that occurs between ISP's and, it determines pretty much the bases for the types of settlements among ISPs:

- backbone ISPs,

- downstream ISPs,
- online service providers,
- ISPs specializing in web hosting.

Backbone ISPs provide connectivity and manage network infrastructure. The four largest backbone ISPs are UUNET, AT&T, SPRINT and GENUITY (Pappalardo, 2001). Since late 90s the large backbone ISPs began changing their interconnection terms. These providers or otherwise called "Tier-1" ISPs have several connections dedicated to inter-connecting their backbones without going through the NAPs. They have increased the amount of "private peering" (SKA settlement) they do between themselves and a few of the other ISPs. The large backbone ISPs agreed to peer (exchange traffic with one another at no cost) only with the other large ISPs and a few other ISPs and have "transit" (exchange traffic for a fee) services with smaller ISPs (Haynal, 2001). The Internet backbone market remains free of Telecommunications Regulation (Kende, 2000), which allows backbone ISPs to make peering decisions freely, without even specifying criteria for peering. Hence the backbone ISPs can choose not to peer or even discriminate between other ISPs in making their peering decisions. This contrasts with other telecommunication industries, where such discrimination is prohibited by regulations. The Backbone ISPs do not form an exclusive category. The backbone ISPs can have also web hosting services or online services (like AT&T and FrenchTelecom). These are referred to as integrated ISPs.

Downstream ISPs serve individuals, businesses and even smaller providers. They pay upstream backbone ISPs for connectivity, the price of which depends on the location and amount of data (Telegeography 2000). Downstream ISPs pay for leasing certain amount of circuits per month as well as a connection fee (unilateral settlement or transit), which lets the downstream ISPs' customers to reach other destinations in the Internet. Most downstream ISPs do not pay based on their actual usage. The payment is based on a usage profile, the overall traffic pattern.

Online service providers, like AOL, earn revenues by providing Internet access, focusing on the content and easiness of use. Online service providers lease connectivity from backbones or other upstream ISPs and manage the network points of presence (POPs) that connect dial-up customers to the Internet. The online service providers are either paid a flat monthly fee by customers for unlimited service or charge additional fees after a certain limit of usage is exceeded. However, much of their revenue comes from selling content and advertising space.

Web hosting companies, like Exodus, host websites that are accessed by the Internet public. It is important to note that the web hosting ISPs create unidirectional traffic, as websites originate a lot of traffic, while not requesting much. As a result, backbone ISPs demand that web hosting providers, which typically do not maintain a national network, purchase connectivity form a backbone or downstream ISPs (Cukier, 1998a).

2.2 Interconnection Arrangements Among Backbone ISPs

In this section we discuss the interconnection arrangements of backbone providers or as we refer them in our paper - Internet Access Providers (IAPs). At the early stages of the Internet the IAPs exchanged traffic mostly at the National Access Points, where each backbone provider had to provide connection only to the NAPs, instead of having individual connections to every other backbone provider. They exchanged traffic under peering arrangements. Such traffic exchange arrangements at the NAPs are called public peering (Kende, 2000). In Figure 1, for instance, backbones 1 and 3 are connected to NAPs in both Washington D.C. and San Francisco, where they can exchange traffic with each other as well as with backbones 2 and 4. On the other hand 2 and 4 are connected to each other and the rest of the backbones only at the NAP in Washington D.C.. It was cost-efficient to provide connections only to NAPs instead of having private connections with each other, due to the cost of large investments in the fiber optic capacity.

As the number of users increased rapidly over the last few years, the NAPs become congested, so users experienced a lot of delays. As a result many large backbones started to interconnect with each other directly through private peering arrangements. For example, backbones 2 and 4 have entered into private peering agreements with each other and can exchange traffic for each other through their private connection in Figure 2, while still using NAP in D.C. to exchange traffic with 1 and 3.

Under the peering agreements backbones 2 and 4 cannot route traffic from their other peering partners through the direct connection they have between themselves,

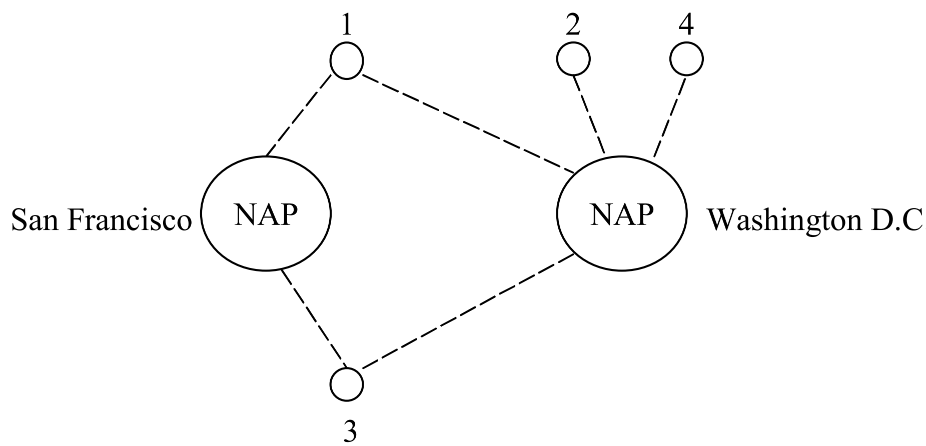

Figure 1:

i.e., 2 will not accept traffic from 4 which is destined to 3 on their private link. However 2 and 4 can exchange traffic of their transit customers on their private link.

Some smaller backbone providers might peer with some backbones while pay transit to other backbones. A few large backbones' interconnections are based entirely on peering arrangements. They do not need to purchase transit.

Currently there is no regulation on interconnection arrangements among backbone providers. Hence the criteria for the peering decisions are not very specific and are made subjectively on case by case. However, several important criteria for peering decisions include geographic spread, capacity, traffic volume and customer profile.

3 Economic Literature on Internet

Most of the economic research on the Internet has focused on pricing and sharing the infrastructure. Mackie-Mason and Varian (1995) propose a smart market mechanism to deal with congestion. They propose to replace current FIFO design with prioritization and to use auctions for congested resources. Odlyzko (1997) suggests multiservice mechanism, where users can choose between the first and second class services and pay accordingly, even though the quality is not necessarily different. Mason (2000) argues that in a duopoly model with overall positive effects flat rate pricing occurs in equilibrium.

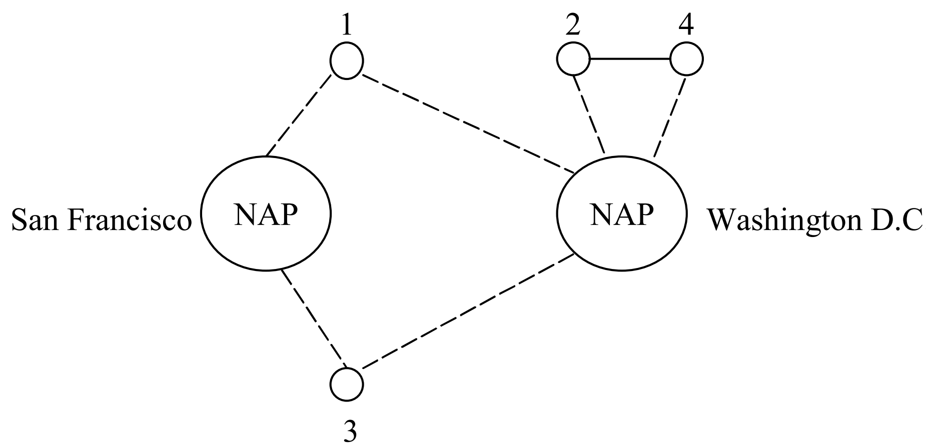

Figure 2:

On the other hand Gibbens et al (2000) discusses the competition between two Internet service providers, when either or both of them choose to offer multiple service classes. Assuming a uniform distribution of user preferences towards congestion, a linear function of congestion and finite number of networks, they prove that, even when Internet service providers are free to set capacities as well as prices, multiproduct competition is not sustainable in a profit maximizing equilibrium. Mason (2001) develops a model where firms are vertically and horizontally differentiated and consumers have different preferences for the firm size and location. He considers a two stage game, where two firms first decide whether to make their goods compatible or not, then they choose prices, given their rival's price. The author concludes that the firms make their goods compatible, the competition increases due to a decrease in vertical differentiation, but at the same time the importance of market share increases, so the competition decreases. The dominance of each effect depends on the relative importance of horizontal and vertical aspects in consumers' utilities.

De Palma and Leruth (1989) discuss a duopoly model, where firms compete on capacities and prices. They consider cases with homogeneous and heterogeneous consumers. They show that competition among firms decreases in the presence of congestion and firms gain monopoly power by selling congested goods. As a result congested goods might be offered at a high price and at a lower quality to consumers.

Cremer et al (2000) describe a model, which analyzes the competition among backbones. The backbones have some installed base of customers, and they compete for new customers. The model incorporates positive externality effects of increasing number of customers. The more customers are attached to the backbone the better is the quality of service. On the other hand the quality of service improves with better interconnection quality of backbones. The demands are based on prices and qualities of service. In the paper, the authors conclude that in the case with backbones of different sizes, the larger backbone prefers a lower quality of interconnection than the smaller backbone. Moreover they show that if the quality of interconnection is costly, perfect connectivity is not efficient socially or privately. In the absence of the cost for the interconnection, the dominant backbone's best strategy is to refuse interconnection with the smaller one. The authors also discuss the case with equal-size backbones. In this case the backbones prefer high quality of interconnection and obtain identical profits at the equilibrium. The results of the case with equal-size backbones are somewhat similar to what we get in our paper, particularly the backbones do prefer higher quality of interconnection, when they serve homogeneous customers, and there are no exogenous assumptions about their sizes.

DangNguyen and Penard (1999) consider a model of vertical differentiation with two asymmetric backbones and identical retail ISPs connected to those backbones. The retail ISPs peer with other ISPs connected to the same backbone (intra-backbone peering) and also with ISPs connected to the other backbone (inter-backbone peering). Intra-backbone peering reduces congestion in that backbone only and raises the quality of all ISPs connected to that backbone. Inter-backbone peering reduces congestion in both backbones and raises quality of both backbones. The authors show that ISPs connected to the high quality backbone will always peer with each other. But they may or may not peer with ISPs connected to the low quality backbone. Also, in the latter case, the ISPs of the low quality backbone will peer with each other. The results are illustrated with some evidence from the French Internet market.

Little and Wright (1999) make a strong case against regulator enforced peering, where regulation forbids payments between access providers as well as their right to refuse to peer. There are two providers whose demands are determined by a model of horizontal product differentiation with a built in asymmetry. Costs include a cost for providing capacity, a fixed cost per customer and a marginal cost of usage. Usage exceeding the capacity is assumed to render zero utility. The authors compare the solution under regulator enforced peering in which firms choose their investments in capacity in the first stage and prices in the second stage, with the welfare maximizing solution in which welfare is measured by the sum of producer and consumer surplus. The former leads to congestion owing to under investment in capacity. If, however, firms peer with settlement payments, namely, net users pay net providers at a rate equal to the marginal cost of providing capacity, the solution obtained is precisely the welfare maximizing one. The same is the case when firms refuse to peer with anyone who is a net user of infrastructure.

Gorman and Malecki (2000) examined the network structure and the performance of ten backbone provider networks in the USA, based on the basic graph theoretic measures together with the median downloading time of those backbone providers. They concluded that even though the basic graph theoretic measures are useful tools, when analyzing the efficiency of the networks, however, they might not be good tools in comparing different networks in one infrastructure. Their analysis show that for an Internet provider network having high graph theoretic measures does not necessarily mean high technical performance. Thus even the complete network, having the highest and most efficient graph-theoretic measures, is not necessarily very efficient Internet network in terms of median downloading time, which is the measure of performance in their paper. To connect its customers to the whole Internet the Internet backbone provider depends on other backbones' networks through public or private peering. The characteristics of certain network and its performance in relation to other networks in the Internet determine the demand for the services of that network.

4 Game Theoretic Analysis

In this section we develop our basic model in the form of a multi-stage game with complete information and solve for the subgame perfect Nash equilibria. There are three separate pieces of analysis. First we discuss a model with no discrimination in the form of a non-cooperative game and find subgame perfect Nash equilibria through backward induction. Then we introduce the possibility of discrimination in the model.

Finally we do network analysis and determine pairwise stable and efficient networks.

Let $N = \{1, 2, ..., n\}$ be a finite set of Internet backbone providers or Internet Access Providers (IAPs) with $n \geq 3$. The network connections among IAPs are represented by undirected links in a graph. The nodes (vertices) of the graphs represent the location of IAPs. All IAPs are connected to National Access Points (NAPs) through which they are connected to other IAPs in the Internet. For simplicity we assume there is only one NAP. We suppose that each IAP is connected to the NAP with a given uniform capacity k, which is the maximum amount of data that can be handled over that link between IAP and NAP at a certain point in time. Thus k is the link capacity of the publicly provided network. IAPs may also decide to enter into private peering agreements with one another.

A link ij is a subset of N that contains i and j. For any two providers, $i \in N$ and $j \in N$, ij refers to the private peering agreement between i and j. The collection of all links on N, $g^n = \{ij \mid i, j \in N, \ i \neq j\}$, is called the complete network on N, where $|g^n| = \frac{n(n-1)}{2}$. In the complete network each IAP has formed private peering agreements with all the other IAPs. Any arbitrary collection of links $g \subset g^n$ is called a network on N. The set of all possible networks on N is denoted by $G = \{g \mid g \subset g^n\}$. $g^0 = \emptyset$ is an empty network, i.e., IAPs connect to each other only through the NAP. The network $g + ij$ where $i, j \notin g$ denotes the new network formed by addition of the link ij to the network g. The network $g - ij$ where $i, j \in g$ denotes the new network formed by removal of the link ij from the network g. Figure 3(a) and 3(b) illustrate the empty network and the complete network, respectively, with three IAPs.

4.1 The Model without Discrimination

We consider a four stage non-cooperative game where at the first stage the IAPs decide whether to have a private peering agreements or not, then decide how much to invest in their own network capacities. If they decide to peer at the first stage, then subsequently they choose investments in link capacities connecting to each other. In the last stage they compete in a' la Bertrand.

Stage 1

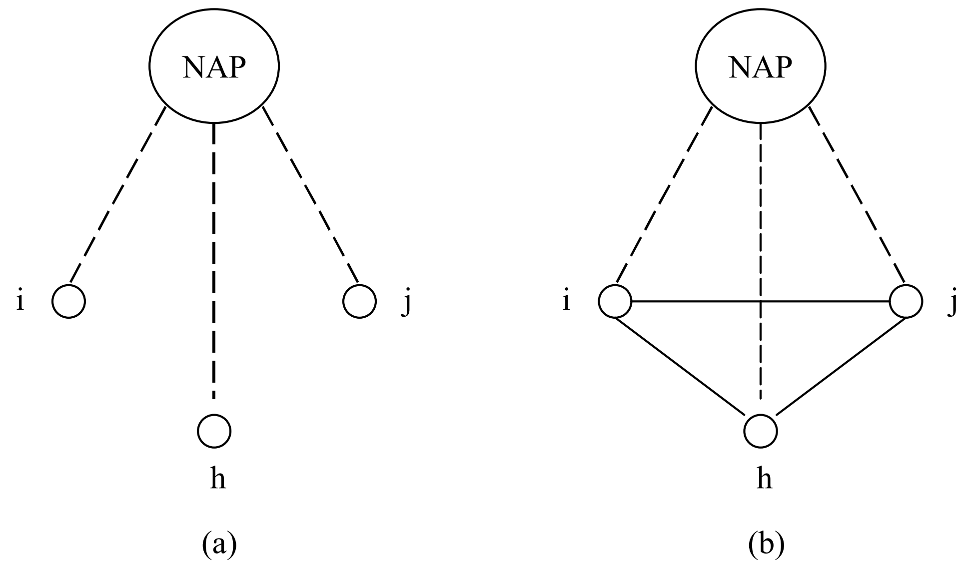

Figure 3:

In this stage each IAP decides to signal its willingness to engage in private peering. Let $\gamma_i = 1$ if i signals to the other IAPs that it is willing to peer with any of the other IAPs and gives permission to other IAPs to build a private link to itself if they want to. γ_i is 0, if it does not want to have any private peerings with other IAPs. Peering agreements materialize between any pair of IAPs, who are willing to peer and give permission to connect to themselves. The peering agreements result in creation of a network $g \in G$.

Stage 2

In this stage IAPs decide how much to invest in the capacities of their own "internal" networks. Each IAP i chooses a capacity level for its network, denoted by s_i. s_i shows how many customers IAP i can serve at a certain point in time. (As we will see later we assume that every customer demands one unit of service).

Stage 3

IAPs choose investments in the links that connect their network to other IAPs' networks with whom peering agreements can be materialized. Let s_i^j denote the investment of IAP i in the link capacity of $ij \in g$ and s_j^i denote the investment of IAP j in the link capacity of ij. Then the capacity of the link ij, k_{ij}, is determined as a sum of s_i^j and s_j^i.

$$k_{ij} = s_i^j + s_j^i$$

We assume that, to connect its customers to IAP j, IAP i uses either the private direct link it has with the server j and/or the NAP. Based on peering arrangements of backbone providers, no backbone provider can transfer traffic from one of its peering partner to another peering partner, i.e., if i has a peering agreement with j and h, then i cannot transfer traffic intended for h which is coming from j (Kende, 2000). Note that even if an IAP is willing to peer and gives permission to connect to itself in the first stage, it still has an option of not making any investments in the private link ij, i.e., $s_j^i \geq 0$ for all $ij \in g$.

Stage 4

In this last stage IAPs compete in prices.

Before we continue with the model we need to define few concepts, which we call connection failure and congestion.

4.1.1 Connection Failure

In our paper we define the connection failure of the IAP i as follows:

$$F_i = \max\{0, d_i - s_i\}$$

where d_i is the demand for IAP i's services. Hence the connection failure of the IAP i depends on its own network's capacity and the demand for its services, namely

$$F_i = \begin{cases} 0, & \text{if } s_i > d_i \\ d_i - s_i, & \text{if } s_i \leqslant d_i \end{cases}$$

The rationale is straight forward. If $s_i > d_i$, then all the customers intending to connect to IAP i can connect. If, however, $s_i \leqslant d_i$, then s_i provides an upper limit to the amount of traffic that can be handled by IAP i and, some customers face connection failures.

Thus there is no connection failure, if the demand does not exceed IAP i's network capacity, i.e., all the customers of IAP i will be connected to IAP i. On the other hand, if the IAP i's demand exceeds its network capacity, then some consumers will not be able to connect to IAP i at all. However, we can rule out the possibility that

$s_i > d_i$. We are assuming that it is costly for IAP i to invest in s_i. So the profit maximizing IAP will not invest in s_i which exceeds its demand. Hence F_i will be reduced to

$$F_i = d_i - s_i$$

Obviously, lower connection failure indicates higher chance of connection and so better service.

4.1.2 Congestion

Even if the customers do not have difficulties connecting to IAPs, they might experience service problems if they want to connect to customers outside IAP i's network. In this paper we introduce a measure of congestion on the link ij.

Assuming the number of customers of IAP i, who want to connect to IAP j's customers is the same as those, who want to connect to any other IAP's customers as well as the customers of IAP i, the total traffic intended for j through i would be $\frac{s_i}{n}$.

On the other hand assume that each IAP uses $\frac{1}{n-1}$ of the publicly provided capacity, k, for connecting to another IAP.[2] Define the congestion of the link as l_{ij} by:

$$l_{ij} = \begin{cases} \frac{s_i}{n} + \frac{s_j}{n} - \min\{\gamma_i, \gamma_j\}(s_i^j + s_j^i) - \frac{k}{n-1}, & \text{if } \frac{s_i}{n} + \frac{s_j}{n} \geq \min\{\gamma_i, \gamma_j\}(s_i^j + s_j^i) + \frac{k}{n-1} \\ 0, & \text{otherwise} \end{cases}$$

where

$$\min\{\gamma_i, \gamma_j\} = \begin{cases} 1, & \text{if } i \text{ and } j \text{ enter into private peering agreements} \\ 0, & \text{otherwise} \end{cases}$$

It is not unrealistic to assume that the number of i's customers who want to connect to any IAP is the same across IAPs. We assume that customers are homogeneous, so IAPs', the large backbone providers', customer profiles are pretty much similar.

Note that if

$$\frac{s_i}{n} + \frac{s_j}{n} < \min\{\gamma_i, \gamma_j\}(s_i^j + s_j^i) + \frac{k}{n-1}$$

[2] As we assume that consumers are homogeneous, then at certain point in time each consumer or consumer's website is equally desirable by all the rest of the consumers in the Internet.

then IAPs invest more than they need to handle the traffic. Consequently we rule out this possibility. Hence

$$l_{ij} = \frac{s_i}{n} + \frac{s_j}{n} - \min\{\gamma_i, \gamma_j\}(s_i^j + s_j^i) - \frac{k}{n-1}$$

Thus there is no congestion on the link ij until the capacity of ij is reached. So the overall congestion factor for the IAP i will be

$$L_i = \sum_{j \in N \backslash \{i\}} l_{ij}$$

The lower the congestion is, the better connected is the IAP.

4.1.3 Consumer Preferences

In our analysis we assume that consumers are homogeneous. Consumers select an IAP through which they want to connect to others on the Internet. When making decisions, consumers consider the prices, connection failure and overall quality of connectivity, i.e., congestion, of IAPs. Each consumer consumes either 0 or 1 unit of service. Let μ and λ represent the weights consumers put on connection failure and congestion, when connecting to the Internet through IAP i. Denote U_i to be the utility of a consumer, who connects to the Internet through IAP i:

$$U_i = V - \mu F_i - \lambda L_i - p_i; \; i = 1, 2, \ldots, n$$

where V,[3] which is assumed constant for all consumers, represents the reservation value of the consumer for connecting to the Internet and p_i is the per unit price charged by IAP i for its services, and $0 < \lambda < \mu \leq 1$, i.e., we assume that customers would prefer to connect even to a congested network than not to connect at all.

4.1.4 Parameter Constraints

In our model we assume

$$0 < s_i \leqslant d_i \text{ and } \frac{s_i}{n} + \frac{s_j}{n} \geqslant \min\{\gamma_i, \gamma_j\}(s_i^j + s_j^i) - \frac{k}{n-1}$$

which holds for values of λ, μ and k given by the shaded area in Figure 4.[4]

[3] We assume V is large enough compared to $\mu F_i + \lambda L_i + p_i$, so that each consumer buys one unit. In this case $U_i^,$s are strictly positive.

[4] Figure 4 is derived by solving for the equlibrium and imposing the aforesaid conditions on the equilibrium values. See appendix.

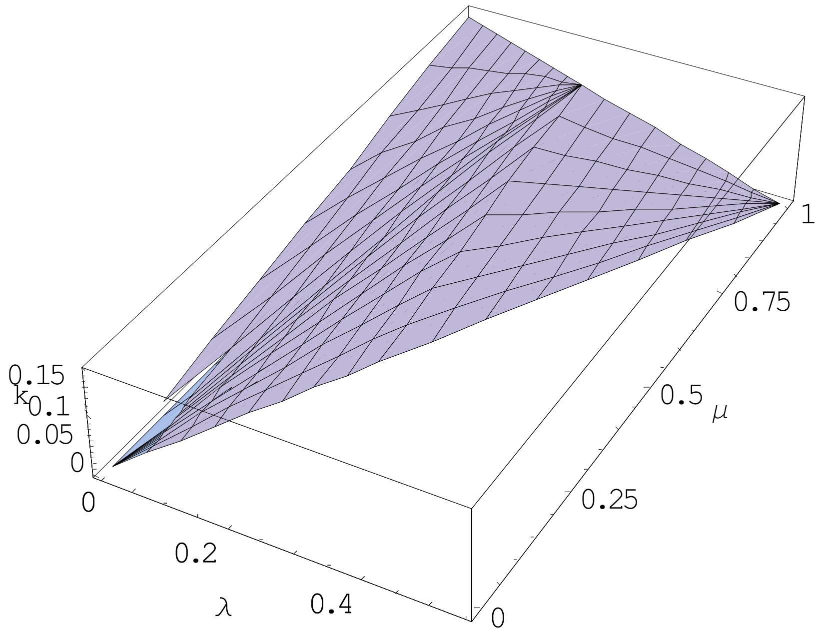

Figure 4:

4.1.5 Equilibrium Analysis

Equilibrium analysis is based on backward induction. Throughout the analysis we assume that the potential market size is normalized to one, i.e., $\sum_{i \in N} d_i = 1$.

We have three cases to consider.

- CASE 1: All IAPs decide to enter into private peering agreements with the rest of IAPs, $\gamma_i = 1$, for all $i \in N$, so we have the complete network, g^n.

- CASE 2: None of the IAPs enter into private peering agreements with the rest of IAPs $\gamma_i = 0$, so we have the empty network, g^0.

- CASE 3: Some of the IAPs enter into private peering agreements, while other do not.

Investments in both server and link capacities are costly for IAPs. We denote costs incurred by IAP i by

$$c_i = s_i^2 + \sum_{j \in N \setminus \{i\}} \min\{\gamma_i, \gamma_j\}(s_i^j)^2$$

Then the profits of IAP i are determined by

$$\pi_i = p_i d_i - c_i$$

Without loss of generality we consider a case with three IAPs, as we can capture essence of all three cases, thus $N = \{1, 2, 3\}$.[5]

Let $\widehat{\boldsymbol{\pi}}_i^l$ be the reduced profit of IAP i, where $i = 1, 2, 3$ in Case l, $l = 1, 2, 3$ in the first stage.

4.1.6 Case 1

In this case all three IAPs decide to enter into private peering agreements with the rest of IAPs, so we have a four stage game. Solving the model by backward induction leads us to a symmetric solution. At the equilibrium all IAPs share the market equally, i.e.,

$$\widehat{d}_i^1 = \frac{1}{3}$$

and charge the same price, which depends only on how much consumers value the capacity of the IAP i's own network. The prices do not depend on the value consumers put on the link capacities, as the link capacities are common for IAPs:

$$\widehat{p}_i^1 = \frac{\mu}{2}.$$

IAPs' optimal investments in link capacities are same for all IAPs and depend on the value the customers put on the interconnection capacities:

$$\widehat{s_i^j}^1 = \frac{\lambda}{15}.$$

The network capacities of IAPs are given by

$$\widehat{s}_i^1 = \frac{2(3\mu - \lambda)}{45} \frac{(75\mu - 4\lambda^2)}{(75\mu - 6\lambda^2)}$$

and they make profits equal to

$$\begin{aligned} \widehat{\pi}_i^1 &= \frac{-1424\lambda^6 + 768\lambda^5\mu - 12\lambda^4\mu(96\mu - 5125) - 28800\lambda^3\mu^2}{36450(2\lambda^2 - 25\mu)^2} \\ &\quad + \frac{1800\lambda^2\mu^2(24\mu - 475) + 270000\lambda\mu^3 - 50625\mu^3(8\mu - 75)}{36450(2\lambda^2 - 25\mu)^2} \\ &= \widehat{\pi}^1 \text{ (say)} \end{aligned}$$

[5] Note that with less than three IAPs, we cannot consider all three cases.

4.1.7 Case 2

In this case when all three IAPs decide not to enter into private peering agreements with the rest of IAPs, we have a three stage game. Solving the model by backward induction we again get a symmetric solution. At the equilibrium all IAPs charge the same price, which is the same as in Case 1, namely

$$\widehat{p}_i^2 = \frac{\mu}{2}$$

and again they share the market equally, i.e.

$$\widehat{d}_i^2 = \frac{1}{3}.$$

Their optimal investments in network capacities are

$$\widehat{s}_i^2 = \frac{2(3\mu - \lambda)}{45}.$$

The profits are given by

$$\widehat{\pi}_i^2 = \frac{-8\lambda^2 + 48\lambda\mu - 72\mu^2 + 675\mu}{4050} = \widehat{\pi}^2 \text{ (say)}.$$

As we can see the prices and demands are the same in both Cases 1 and 2, i.e., $\widehat{p}_i^2 = \widehat{p}_i^1$ and $\widehat{d}_i^2 = \widehat{d}_i^1$. When we compare the investments in network capacities, we can see that the investments are higher in the case with no private peering than that in the case with private peering, namely

$$\frac{2(3\mu - \lambda)}{45}\frac{(75\mu - 4\lambda^2)}{(75\mu - 6\lambda^2)} > \frac{2(3\mu - \lambda)}{45}$$

as $0 < \lambda < \mu \leq 1$.

On the other hand in the case with no private peering the IAPs do not have to invest in link capacities at all. So the profits in the case with no private peerings is unambiguously higher than that in the case with private peerings, i.e., for all values of λ and μ satisfying $0 < \lambda < \mu \leq 1$,

$$\widehat{\boldsymbol{\pi}}^2 > \widehat{\boldsymbol{\pi}}^1$$

Consequently we get the following proposition.

Proposition 1 *If $0 < \lambda < \mu \leq 1$, profits are higher in the network g^0, where none of the IAPs enter into private peering agreements (Case 2), than in the network g^n, where all of them do (Case 1).*

The intuition is straightforward. Given the symmetry of the model, no additional demand can be obtained with the same prices by competing for customers through engaging in private peering. Hence private peering basically results in additional costs through additional investments in both network and link capacities.

4.1.8 Case 3

In Case 3 two of the IAPs enter into private peering agreement, while the third one does not. Then the prices, market shares and both network and link investments of those IAPs that peer are the same. Their profits are also identical and given by $\widehat{\boldsymbol{\pi}}^{3'}$ (say). Let the profits of the non-peering IAP be given by $\widehat{\boldsymbol{\pi}}^{3}$. All the equilibrium values for this case are given in Appendix B.

4.1.9 Subgame Perfect Nash Equilibrium

We can represent the first stage in the following normal form.

2 peers ($\gamma_2 = 1$)

1\3	peer ($\gamma_3 = 1$)	not peer ($\gamma_3 = 0$)
peer ($\gamma_1 = 1$)	$\widehat{\boldsymbol{\pi}}^{1}, \widehat{\boldsymbol{\pi}}^{1}, \widehat{\boldsymbol{\pi}}^{1}$ *	$\widehat{\boldsymbol{\pi}}^{3'}, \widehat{\boldsymbol{\pi}}^{3'}, \widehat{\boldsymbol{\pi}}^{3}$
not peer ($\gamma_1 = 0$)	$\widehat{\boldsymbol{\pi}}^{3}, \widehat{\boldsymbol{\pi}}^{3'}, \widehat{\boldsymbol{\pi}}^{3'}$	$\widehat{\boldsymbol{\pi}}^{2}, \widehat{\boldsymbol{\pi}}^{2}, \widehat{\boldsymbol{\pi}}^{2}$

2 does not peer ($\gamma_2 = 0$)

1\3	peer ($\gamma_3 = 1$)	not peer ($\gamma_3 = 0$)
peer ($\gamma_1 = 1$)	$\widehat{\boldsymbol{\pi}}^{3'}, \widehat{\boldsymbol{\pi}}^{3}, \widehat{\boldsymbol{\pi}}^{3'}$	$\widehat{\boldsymbol{\pi}}^{2}_{1}, \widehat{\boldsymbol{\pi}}^{2}_{2}, \widehat{\boldsymbol{\pi}}^{2}_{3}$
not peer ($\gamma_1 = 0$)	$\widehat{\boldsymbol{\pi}}^{2}, \widehat{\boldsymbol{\pi}}^{2}, \widehat{\boldsymbol{\pi}}^{2}$	$\widehat{\boldsymbol{\pi}}^{2}, \widehat{\boldsymbol{\pi}}^{2}, \widehat{\boldsymbol{\pi}}^{2}$*

It is the case that for all values of λ and μ satisfying $0 < \lambda < \mu \leq 1$,

$$\widehat{\boldsymbol{\pi}}^{3'} > \widehat{\boldsymbol{\pi}}^{2} > \widehat{\boldsymbol{\pi}}^{1} > \widehat{\boldsymbol{\pi}}^{3} \quad (1)$$

Thus, in Case 3 the IAPs that peer earn higher profits compared to the Case 2, the empty network. On the other hand the IAP that does not allow peering in Case 3 earns lower profit compared to the profits it can get in both the empty network and the complete network. The IAPs engaging in private peering offer lower connection failures and less congestion in Case 3, as they invest in link capacities compared to the empty network, and capture a larger share of the market at the expense of the IAP, which is not peering.[6] Consequently the complete network is consistent with a Nash equilibrium. Needless to say, the empty network is also a Nash equilibrium. The subgame perfect Nash equilibria are indicated by an asterisk This is summarized in the following proposition.

Proposition 2 *If* $0 < \lambda < \mu \leq 1$ *and* $k \leq 0.15$, *then there are two subgame perfect Nash equilibria: the complete network,* g^n*, which results in private peering agreements among all the IAPs, and the empty network* g^0 *The empty network is Pareto superior to the complete network.*

In the case with only two players, one can easily show that the subgame perfect Nash equilibrium would not involve any peering.[7] However, in the three player case, one can find an equilibrium with peering.

Example 3 *Let* $\lambda = 0.4$ *and* $\mu = 0.9$ *and* $k = 0.02$. *Then the profits are as follows*

$$\begin{aligned} \widehat{\boldsymbol{\pi}}^1 &= 0.138028 \\ \widehat{\boldsymbol{\pi}}^2 &= 0.139551 \\ \widehat{\boldsymbol{\pi}}^{3'} &= 0.141747 \\ \widehat{\boldsymbol{\pi}}^3 &= 0.133701 \end{aligned}$$

4.2 The Role of Public Infrastructure

In this section we will examine the role of public infrastructure, denoted by k in the model. Without private peering congestion on the line connecting i and j is $\frac{s_i}{3} + \frac{s_j}{3} - \frac{k}{2}$. Given that at the equilibrium $s_i + s_j \leq \frac{1}{3}$, if $k > \frac{4}{9}$, there is no congestion on the National Access grid, even without peering. The obvious impact of this is that

[6] Computations are available at the following link: www.filebox.vt.edu/users/schakrab/computations.htm

[7] In fact if forced to peer, they will not invest anything in link capacities because of the classic "tragedy of the commons" argument. Little and Wright (2000) show this using an elaborate model of horizontal product differentiation.

all incentives for private peering are eliminated and IAPs compete only in network capacities and prices. Consequently, the model is reduced to a two stage game in which IAPs first choose their networks capacities and then choose prices. We will call this Case 5.

We have solved for the subgame perfect Nash equilibrium using backward induction and obtained the following values:

$$\widehat{s}_i^5 = \frac{2\mu}{15}$$

$$\widehat{p}_i^5 = \frac{\mu}{2}$$

$$\widehat{d}_i^5 = \frac{1}{3}$$

$$\widehat{\pi}_i^5 = \frac{\mu(75 - 8\mu)}{450}$$

It is important to note that for all values of $0 < \lambda < \mu \leq 1$, profitability is unambiguously lower in Case 5 compared to Case 1 (and Case 2). Hence we get the following proposition.

Proposition 4 *If the investment in public infrastructure is sufficiently large, i.e., $k > \frac{4}{9}$, then in the unique SPNE IAPs do not peer and profits are unambiguously lower compared to those in the complete and empty networks with congestion at the NAP.*

The reason is that even though firms do not invest in link capacities, investment in network capacity is much higher. It shows why increases in publicly provided infrastructure may not be in the best interests of firms and IAPs may lobby against such increases.

4.3 Consumer Welfare Analysis

We finally compare consumer utilities across the different cases. First note that utility is higher in Case 1 compared to Case 2. This is because while demands and prices are the same, investments in the network capacities are higher in Case 1 compared to Case 2. Also, investments in the link capacities are positive in Case 1 and zero in Case 2. Thus the connection failure and congestion are lower in Case 1 than in Case

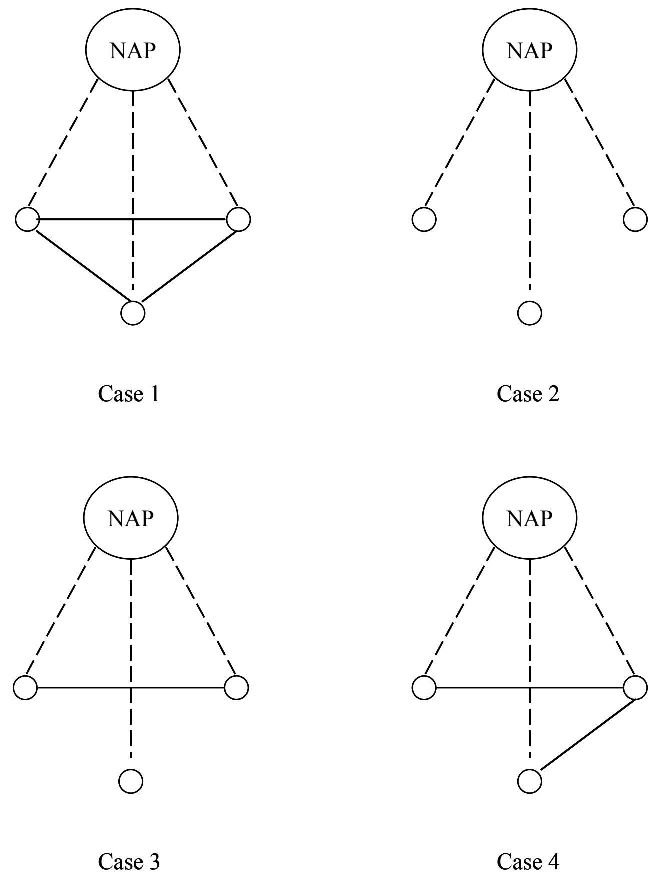

Figure 5:

2. So we can say that the subgame perfect Nash equilibrium results in an efficient outcome from the consumers' point of view.

Next we compare Case 1 and Case 5. Investments in network capacities are higher in Case 5 compared to Case 1. Also, while there is some congestion in Case 1, there is zero congestion in Case 5. Given that demands and prices are the same, utility of the consumer is higher in Case 5 relative to Case 1.

Large publicly provided infrastructure benefits consumers and hurts providers. Hence whether such infrastructure would be provided depends on the relative lobbying power of the IAPs vs. consumer groups.

4.4 The Model with Discrimination

Currently, unlike some other areas of telecommunications, there are no regulations prohibiting discrimination between IAPs with regard to peering. In the basic model, any IAP who wants to enter into peering agreements must do so with all other IAPs who are willing to peer as well. In this section we modify the model, where we take into account the possibility that an IAP may choose to peer with one but not the other IAP. Stages 2, 3 and 4 remain completely unchanged. However we redefine the strategies in the first stage as follows.

Let γ_{ij} be 1 if i signals its willingness to peer with j and 0, if it does not. Peering agreements materialize between any pair of IAPs, who are willing to give permission to connect to each other. The peering agreements result in a creation of a network $g \in G$. Then four possible network configurations are possible. These are represented in Figure 5. We have already analyzed cases 1, 2 and 3. Hence we have to analyze one additional case namely Case 4.

4.4.1 Case 4

In Case 4 two of the IAPs do not enter into mutual private peering agreement, but they both peer with the third IAP. Then the prices, market shares and both network and link investments and profits of those IAPs that do not peer with each other are the same. All the equilibrium values for this case are given in the appendix. If i and j don't peer with each other but both peer with h, then let the reduced profits of i and j be denoted by are $\widehat{\boldsymbol{\pi}}^4$ respectively, while the profit of h is $\widehat{\boldsymbol{\pi}}^{4'}$.

4.4.2 Subgame Perfect Nash Equilibrium

It is the case that for all values of λ and μ satisfying $0 < \lambda < \mu \leq 1$,[8](refer to Appendix C)

$$\widehat{\boldsymbol{\pi}}^{4'} > \widehat{\boldsymbol{\pi}}^{3'} > \widehat{\boldsymbol{\pi}}^{2} > \widehat{\boldsymbol{\pi}}^{1} > \widehat{\boldsymbol{\pi}}^{4} > \widehat{\boldsymbol{\pi}}^{3} \tag{2}$$

[8]Computations are available at the following link: www.filebox.vt.edu/users/schakrab/computations.htm

We can represent the first stage in the following normal form.

2

$\gamma_{21}=\gamma_{23}=1$

1/3	$\gamma_{31}=\gamma_{32}=1$	$\gamma_{31}=1,\gamma_{32}=0$	$\gamma_{31}=0,\gamma_{32}=1$	$\gamma_{31}=\gamma_{32}=0$
$\gamma_{12}=\gamma_{13}=1$	$\widehat{\pi}^{1},\widehat{\pi}^{1},\widehat{\pi}^{1}$ *	$\widehat{\pi}^{4'},\widehat{\pi}^{4},\widehat{\pi}^{4}$	$\widehat{\pi}^{4},\widehat{\pi}^{4'},\widehat{\pi}^{4}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3'},\widehat{\pi}^{3}$
$\gamma_{12}=1,\gamma_{13}=0$	$\widehat{\pi}^{4},\widehat{\pi}^{4'},\widehat{\pi}^{4}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3'},\widehat{\pi}^{3}$	$\widehat{\pi}^{4},\widehat{\pi}^{4'},\widehat{\pi}^{4}$ *	$\widehat{\pi}^{3'},\widehat{\pi}^{3'},\widehat{\pi}^{3}$
$\gamma_{12}=0,\gamma_{13}=1$	$\widehat{\pi}^{4},\widehat{\pi}^{4},\widehat{\pi}^{4'}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3},\widehat{\pi}^{3'}$	$\widehat{\pi}^{3},\widehat{\pi}^{3'},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$
$\gamma_{12}=\gamma_{13}=0$	$\widehat{\pi}^{3},\widehat{\pi}^{3'},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{3},\widehat{\pi}^{3'},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$

2

$\gamma_{21}=1,\gamma_{23}=0$

1/3	$\gamma_{31}=\gamma_{32}=1$	$\gamma_{31}=1,\gamma_{32}=0$	$\gamma_{31}=0,\gamma_{32}=1$	$\gamma_{31}=\gamma_{32}=0$
$\gamma_{12}=\gamma_{13}=1$	$\widehat{\pi}^{4'},\widehat{\pi}^{4},\widehat{\pi}^{4}$	$\widehat{\pi}^{4'},\widehat{\pi}^{4},\widehat{\pi}^{4}$ *	$\widehat{\pi}^{3'},\widehat{\pi}^{3'},\widehat{\pi}^{3}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3'},\widehat{\pi}^{3}$
$\gamma_{12}=1,\gamma_{13}=0$	$\widehat{\pi}^{3'},\widehat{\pi}^{3'},\widehat{\pi}^{3}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3'},\widehat{\pi}^{3}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3'},\widehat{\pi}^{3}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3'},\widehat{\pi}^{3}$ *
$\gamma_{12}=0,\gamma_{13}=1$	$\widehat{\pi}^{3'},\widehat{\pi}^{3},\widehat{\pi}^{3'}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$
$\gamma_{12}=\gamma_{13}=0$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$

2

$\gamma_{21}=0,\gamma_{23}=1$

1/3	$\gamma_{31}=\gamma_{32}=1$	$\gamma_{31}=1,\gamma_{32}=0$	$\gamma_{31}=0,\gamma_{32}=1$	$\gamma_{31}=\gamma_{32}=0$
$\gamma_{12}=\gamma_{13}=1$	$\widehat{\pi}^{4},\widehat{\pi}^{4},\widehat{\pi}^{4'}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3},\widehat{\pi}^{3'}$	$\widehat{\pi}^{3},\widehat{\pi}^{3'},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$
$\gamma_{12}=1,\gamma_{13}=0$	$\widehat{\pi}^{3},\widehat{\pi}^{3'},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{3},\widehat{\pi}^{3'},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$
$\gamma_{12}=0,\gamma_{13}=1$	$\widehat{\pi}^{4},\widehat{\pi}^{4},\widehat{\pi}^{4'}$ *	$\widehat{\pi}^{3'},\widehat{\pi}^{3},\widehat{\pi}^{3'}$	$\widehat{\pi}^{3},\widehat{\pi}^{3'},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$
$\gamma_{12}=\gamma_{13}=0$	$\widehat{\pi}^{3},\widehat{\pi}^{3'},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{3},\widehat{\pi}^{3'},\widehat{\pi}^{3'}$ *	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$

2

$\gamma_{21}=\gamma_{23}=0$

1/3	$\gamma_{31}=\gamma_{32}=1$	$\gamma_{31}=1,\gamma_{32}=0$	$\gamma_{31}=0,\gamma_{32}=1$	$\gamma_{31}=\gamma_{32}=0$
$\gamma_{12}=\gamma_{13}=1$	$\widehat{\pi}^{3'},\widehat{\pi}^{3},\widehat{\pi}^{3'}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3},\widehat{\pi}^{3'}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$
$\gamma_{12}=1,\gamma_{13}=0$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$
$\gamma_{12}=0,\gamma_{13}=1$	$\widehat{\pi}^{3'},\widehat{\pi}^{3},\widehat{\pi}^{3'}$	$\widehat{\pi}^{3'},\widehat{\pi}^{3},\widehat{\pi}^{3'}*$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$
$\gamma_{12}=\gamma_{13}=0$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}$	$\widehat{\pi}^{2},\widehat{\pi}^{2},\widehat{\pi}^{2}*$

For instance, if $\lambda = 0.4$ and $\mu = 0.9$ and $k = 0.02$. Then the profits are as follows

$$\begin{aligned}
\widehat{\pi}^{4'} &= 0.144 \\
\widehat{\pi}^{3'} &= 0.142 \\
\widehat{\pi}^{2} &= 0.140 \\
\widehat{\pi}^{1} &= 0.138 \\
\widehat{\pi}^{4} &= 0.136 \\
\widehat{\pi}^{3} &= 0.134
\end{aligned}$$

Then one can easily verify from the above matrices that there are eight subgame perfect Nash equilibria. In fact, all cases, Case 1, Case 2, Case3 and Case 4 are consistent with a subgame perfect Nash equilibrium. We have indicated all the equilibria by an asterisk

Proposition 5 *Under conditions* $0 < \lambda < \mu \leq 1$ *if we allow the possibility of discrimination with regard to peering, besides the complete network and the empty network, any incomplete network is also consistent with a subgame perfect Nash equilibrium.*

Unlike certain other telecommunication industries, large backbones are not regulated by any sort of regulatory framework. Hence discrimination is a realistic assumption. The surfeit of subgame perfect Nash equilibria make it imperative to use other equilibrium concepts to figure out plausible equilibrium configurations.

5 Network Analysis

Finally, we perform a formal network analysis. Peering is a collaborative effort. Hence, noncooperative games cannot fully analyze such a setting in its full complexity.

A network is useful tool to analyze such an effort, because it combines elements of both cooperative and non-cooperative game theory.

The first stage is redefined as a link formation stage. A link $ij \in g$ indicates collaboration on form of a peering agreement. The emphasis is on stability. A stable network is one in which IAPs want to maintain existing links but do not want to form new ones. This is captured by the concept of pairwise stability.

The concept of pairwise stability was introduced by Jackson and Wolinsky (1996) in the context of normal form network formation. We modify this definition in the set up of an extensive form network formation.

Definition 6 *Let $\vartheta_i(g)$ denote the reduced profits in Stage 1 for a network $g \in G$. Then the network g is pairwise stable if*

$$\begin{aligned} &\text{(a) for all } ij \in g,\ \vartheta_i(g) \geqslant \vartheta_i(g-ij) \text{ and } \vartheta_j(g) \geqslant \vartheta_j(g-ij) \\ &\text{(b) for all } ij \notin g,\ \text{if } \vartheta_i(g) < \vartheta_i(g+ij), \text{ then } \vartheta_j(g) > \vartheta_j(g+ij) \end{aligned}$$

The next definition is one of efficiency. In our model, an efficient network is one which maximizes joint profits. Hence, formally,

Definition 7 *Let $\vartheta_i(g)$ denote the reduced profits in Stage 1 for a network $g \in G$. Let $W(g) = \sum_{i \in N} \vartheta_i(g)$. A network g is efficient if $W(g) \geqslant W(g')$ for all $g' \in G$.*

Stages 2, 3 and 4 essentially remain unchanged. Next we come to the main result of our paper.

Theorem 8 *(a) The unique pairwise stable network is the complete network g^n. (b) The unique efficient network is the empty network g^0.*

Proof. (a) We will consider each case with three IAPs.

Case 1: Consider the complete network g^3. Each IAP i earns a profit of $\vartheta_i(g^3) = \widehat{\boldsymbol{\pi}}^1$. Each IAP has private peering agreements with all the rest of IAPs. So there are no links to form. On the other hand if i and j delete their mutual link, they earn $\vartheta_i(g^3 - ij) = \vartheta_j(g^3 - ij) = \widehat{\boldsymbol{\pi}}^4$. Since, $\widehat{\boldsymbol{\pi}}^1 > \widehat{\boldsymbol{\pi}}^4$, $\vartheta_i(g^3 - ij) < \vartheta_i(g^3)$ and $\vartheta_j(g^3 - ij) < \vartheta_j(g^3)$. Hence, the complete network is pairwise stable.

The rest of the proof consists of showing that no other network architecture is pairwise stable.

Case 2: Consider an empty network g^0. Each IAP earns a profit given by $\vartheta_i(g^0) = \widehat{\boldsymbol{\pi}}^2$. There are no links to delete. On the other hand if i and j form a link, they earn $\vartheta_i(g^0 + ij) = \vartheta_j(g^0 + ij) = \widehat{\boldsymbol{\pi}}^{3'}$. Since $\widehat{\boldsymbol{\pi}}^{3'} > \widehat{\boldsymbol{\pi}}^2$, $\vartheta_i(g^0 + ij) < \vartheta_i(g^0)$, and $\vartheta_j(g^0 + ij) < \vartheta_j(g^0)$. Hence the empty network is not pairwise stable.

Case 3: Consider the network represented by Case 3, say, $g = \{12\}$. Then the payoffs of the IAPs are $\vartheta_1(g) = \vartheta_2(g) = \widehat{\boldsymbol{\pi}}^{3'}$, $\vartheta_3(g) = \widehat{\boldsymbol{\pi}}^3$. If 1 and 3 form a link, then 1 earns $\vartheta_1(g + 13) = \widehat{\boldsymbol{\pi}}^{4'}$ and 3 earns $\vartheta_3(g + 13) = \widehat{\boldsymbol{\pi}}^4$. Now $\widehat{\boldsymbol{\pi}}^{4'} > \widehat{\boldsymbol{\pi}}^{3'}$ and $\widehat{\boldsymbol{\pi}}^4 > \widehat{\boldsymbol{\pi}}^3$. Hence, $\vartheta_1(g+13) > \vartheta_1(g)$ and $\vartheta_3(g+13) > \vartheta_3(g)$. Hence, the network is not pairwise stable.

Case 4: Consider the network represented by Case 4, say, $g = \{13, 12\}$. Then the payoffs of the IAPs are $\vartheta_3(g) = \vartheta_2(g) = \widehat{\boldsymbol{\pi}}^4$, $\vartheta_1(g) = \widehat{\boldsymbol{\pi}}^{4'}$. If 2 and 3 form a link, then 2 earns $\vartheta_2(g+23) = \widehat{\boldsymbol{\pi}}^1$ and 3 earns $\vartheta_3(g+23) = \widehat{\boldsymbol{\pi}}^1$. As $\widehat{\boldsymbol{\pi}}^1 > \widehat{\boldsymbol{\pi}}^4$, $\vartheta_2(g+23) > \vartheta_2(g)$ and $\vartheta_3(g + 23) > \vartheta_3(g)$. Hence, the network is not pairwise stable.

That completes the proof.

(b) First we can show that for all values of λ and μ satisfying $0 < \lambda < \mu \leq 1$,

$$3\ \widehat{\boldsymbol{\pi}}^2 > 2\widehat{\boldsymbol{\pi}}^{3'} + \widehat{\boldsymbol{\pi}}^3 > \widehat{\boldsymbol{\pi}}^{4'} + 2\ \widehat{\boldsymbol{\pi}}^4 > 3\ \widehat{\boldsymbol{\pi}}^1 \tag{3}$$

Hence joint profits are strictly decreasing in the number of links. Consequently the result follows.[9] ■

Hence we arrive at the result that stable networks are not efficient and vice versa. The intuition is straight forward. Each link benefits the IAPs forming the link at the expense of the third IAP. But the gain through the link formation is more than offset by the loss to the third party. The large number of equilibria in a noncooperative game setting is drastically reduced in a network setting. This is because Nash equilibria focus on individual deviations. On the contrary, pairwise stability focusses on pairwise deviations. Needless, to say the latter result is a stronger condition than the former.

[9] Computations are available at the following link: www.filebox.vt.edu/users/schakrab/computations.htm

6 Conclusion

We find that in this relatively simple model, where demand is fixed, and consumers do not drop out with the declining quality of service, there is still a case for peering. When we incorporate network externalities, and the consumer participation constraint, the case for peering will be even stronger for congested NAPs. We further show that a congested NAP is not necessarily a bad thing as far as IAPs are concerned, because it increases their profits by opening up the possibility of peering.

The paper enables comparison of pure non-cooperative game theoretic set-up with a networks set-up, which combines elements of both cooperative and non-cooperative game theory. Given that, in fact, there is extensive private peering among large backbones, it follows that the network approach results in both stronger and more realistic conclusions.[10] Hence this paper illustrates the advantages of using a mixed approach over a purely non-cooperative approach.

[10] From a purely theoretical perspective, in general, Nash equilibria are not comparable to pairwise stable network configurations. There can be Nash equilibria resulting in networks that are not pairwise stable and pairwise stable networks that are not consistent with Nash equilibria.

Appendix A

Given a network g, we first illustrate how we have solved stages 2,3 and 4 using backward induction. Assume that values of the primitives of the model λ, μ and k lie in the area shown by Figure 4.

Determining demand

Denote the demand for IAP i by d_i. Then in equilibrium, a consumer is indifferent between the three IAPs if

$$\mu F_1 + \lambda L_1 + p_1 = \mu F_2 + \lambda L_2 + p_2 = \mu F_3 + \lambda L_3 + p_3$$

where $F_i = d_i - s_i$

This gives us equilibrium values of d_i in terms of L_i, p_i and s_i say d_i^*

Solving for prices

Profits for IAP i are given by

$$\pi_i = p_i \cdot d_i^* - c_i$$

Hence, our first order conditions are given by

$$\frac{\partial \left(p_i \cdot d_i^* - c_i\right)}{\partial p_i} = 0,\ i = 1, 2, 3$$

which gives us a linear system of three simultaneous linear equations with three unknowns, which can be solved to obtain the equilibrium prices.

Next we plug in the equilibrium prices, p_i^*, and the values of L_is to obtain the reduced form of profits in terms of network capacities and link capacities. Note that the value of L_i will vary from one network to another. Define a variable q_{ij}, $j \neq i$ to be equal to 1, if there is a peering arrangement between i and j in the network g and zero otherwise. Then,

$$L_i = \frac{s_i}{3} + \frac{s_j}{3} - q_{ij} \cdot (s_i^j + s_j^i) - \frac{k}{2} + \frac{s_i}{3} + \frac{s_h}{3} - q_{ih} \cdot (s_i^h + s_h^i) - \frac{k}{2}$$

where $h \neq i, j \neq i$.

Solving for link capacities

This stage is relevant if and only if there is some peering i.e. $q_{ij} = 1$ for some $i, j \in N, i \neq j$. The first order conditions are given by

$$\frac{\partial \pi_i}{\partial s_{ij}} = 0$$

This gives us $\sum_{i \in N} \sum_{j \neq i} q_{ij}$ simultaneous linear equations in an equal number of unknowns which can be solved to obtain optimal link capacities. We plug in the optimal link capacities in the reduced profits for the third stage to obtain reduced profits for the second stage in terms of the network capacity only.

Solving for network capacities

Finally, we solve for network capacities. The first order conditions are given by

$$\frac{\partial \pi_i}{\partial s_i} = 0$$

This gives us three linear equations in three unknowns which can be solved to obtain the equilibrium network capacities. We plug in the optimal network capacity in the reduced profits for the second stage to obtain reduced profits for the first stage which we denote by $\widehat{\pi}_i^l$ for IAP i, $i = 1, 2, 3$ for case l, $l = 1, 2, 3, 4, 5$.

Next we show the results for Cases 3 and 4. We show them separately in the appendix because the expressions are too cumbersome to be included in the main body of the article.

Appendix B

Case 3

Assume i and j peer with each other while h does not peer with anyone. Then the equilibrium prices in the subgame perfect Nash equilibrium are given by

$$\widehat{p}_i^3 = \frac{5625\mu^3(16\lambda^2 - 24\lambda\mu + 9\mu(4\mu - 25))}{2(8\lambda^6 - 48\lambda^5\mu + 3000\lambda^3\mu^2 - 4500\lambda^2(\mu - 35)\mu^2 - 135000\lambda\mu^3 + 50265\mu^3(4\mu - 25) + 4\lambda^4\mu(18\mu - 1025))}$$

$$\widehat{p}_j^3 = \frac{5625\mu^3(16\lambda^2 - 24\lambda\mu + 9\mu(4\mu - 25))}{2(8\lambda^6 - 48\lambda^5\mu + 3000\lambda^3\mu^2 - 4500\lambda^2(\mu - 35)\mu^2 - 135000\lambda\mu^3 + 50265\mu^3(4\mu - 25) + 4\lambda^4\mu(18\mu - 1025))}$$

$$\widehat{p}_h^3 = \frac{3\mu(8\lambda^6 - 48\lambda^5\mu + 3000\lambda^3\mu^2 - 45000\lambda\mu^3 - 1500\lambda^2\mu^2(3\mu - 65) + 16875\mu^3(4\mu - 25) + 4\lambda^4\mu(18\mu - 1025))}{2(8\lambda^6 - 48\lambda^5\mu + 3000\lambda^3\mu^2 - 4500\lambda^2(\mu - 35)\mu^2 - 135000\lambda\mu^3 + 50265\mu^3(4\mu - 25) + 4\lambda^4\mu(18\mu - 1025))}$$

The demands in equilibrium are given by

$$\widehat{d}_i^3 = \frac{1875\mu^3(16\lambda^2 - 24\lambda\mu + 9\mu(4\mu - 25))}{2(8\lambda^6 - 48\lambda^5\mu + 3000\lambda^3\mu^2 - 4500\lambda^2(\mu - 35)\mu^2 - 135000\lambda\mu^3 + 50265\mu^3(4\mu - 25) + 4\lambda^4\mu(18\mu - 1025))}$$

$$\widehat{d}_j^3 = \frac{1875\mu^3(16\lambda^2-24\lambda\mu+9\mu(4\mu-25))}{2(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))}$$

$$\widehat{d}_h^3 = \frac{8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-45000\lambda\mu^3-1500\lambda^2\mu^2(3\mu-65)+16875\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025)}{2(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))}$$

Their optimal investments in network capacities are

$$\widehat{s}_i^3 = \frac{10(\lambda-3\mu)\mu(32\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)-375\lambda^2\mu^2(12\mu-155)+2\lambda^4\mu(36\mu-1225))}{(4\lambda^2-75\mu)(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))}$$

$$\widehat{s}_j^3 = \frac{10(\lambda-3\mu)\mu(32\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)-375\lambda^2\mu^2(12\mu-155)+2\lambda^4\mu(36\mu-1225))}{(4\lambda^2-75\mu)(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))}$$

$$\widehat{s}_h^3 = \frac{10(\lambda-3\mu)\mu(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-45000\lambda\mu^3-1500\lambda^2\mu^2+16875\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))}{(4\lambda^2-75\mu)(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))}$$

The optimal investments in link capacities are

$$\widehat{s_i^j}^3 = \frac{375\lambda\mu^2(16\lambda^2-24\lambda\mu+9\mu(4\mu-25))}{(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))}$$

$$\widehat{s_j^i}^3 = \frac{375\lambda\mu^2(16\lambda^2-24\lambda\mu+9\mu(4\mu-25))}{(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))}$$

The profits are given by

$$\widehat{\pi}_i^3 = \left(\frac{-(25\mu^2(16\lambda^2-24\lambda\mu+9\mu(4\mu-25))^2)}{(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))^2}\right) \left(\frac{(32\lambda^{10}-192\lambda^9\mu+24000\lambda^7\mu^2-1110000\lambda^5\mu^3+22500000\lambda^3\mu^4-168750000\lambda\mu^5+(31640625\mu^5(8\mu-75)}{2(4\lambda^2-75\mu)^2} + \frac{32\lambda^8\mu(9\mu-125)-1406250\lambda^2\mu^4(24\mu-245)-1000\lambda^6\mu^2(36\mu-365))+15000\lambda^4\mu^3(111\mu-1150))}{2(4\lambda^2-75\mu)^2}\right)$$

$$\widehat{\pi}_j^3 = \left(\frac{-(25\mu^2(16\lambda^2-24\lambda\mu+9\mu(4\mu-25))^2)}{(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))^2}\right) \left(\frac{(32\lambda^{10}-192\lambda^9\mu+24000\lambda^7\mu^2-1110000\lambda^5\mu^3+22500000\lambda^3\mu^4-168750000\lambda\mu^5+(31640625\mu^5(8\mu-75)}{2(4\lambda^2-75\mu)^2} + \frac{32\lambda^8\mu(9\mu-125)-1406250\lambda^2\mu^4(24\mu-245)-1000\lambda^6\mu^2(36\mu-365))+15000\lambda^4\mu^3(111\mu-1150))}{2(4\lambda^2-75\mu)^2}\right)$$

$$\widehat{\pi}_h^3 = \left(\frac{8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-45000\lambda\mu^3-1500\lambda^2\mu^2(3\mu-65)+16875\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025)^2}{(8\lambda^6-48\lambda^5\mu+3000\lambda^3\mu^2-4500\lambda^2(\mu-35)\mu^2-135000\lambda\mu^3+50265\mu^3(4\mu-25)+4\lambda^4\mu(18\mu-1025))^2}\right) \left(\frac{\mu(48\lambda^4-2000\lambda^2\mu+1200\lambda\mu^2-225\mu^2(-75+8\mu))}{2(4\lambda^2-75\mu)^2}\right)$$

Appendix C

Case 4

Assume i and j peer with each other, so do i and h. But j and h do not peer. Then the equilibrium prices in the subgame perfect Nash equilibrium are given by

$$\widehat{p}_i^4 = \frac{-3\mu(64\lambda^6+48\lambda^5\mu-3000\lambda^3\mu^2+45000\lambda\mu^3-16875\mu^3(4\mu-25)+750\lambda^2\mu^2(6\mu-25)-8\lambda^4\mu(9\mu+275))}{2(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

$$\widehat{p}_j^4 = \frac{3\mu(104\lambda^6-195\lambda^5\mu+6000\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)-750\lambda^2\mu^2(12\mu-115)+4\lambda^4\mu(72\mu-1375)}{2(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

$$\widehat{p}_h^4 = \frac{3\mu(104\lambda^6-195\lambda^5\mu+6000\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)-750\lambda^2\mu^2(12\mu-115)+4\lambda^4\mu(72\mu-1375)}{2(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

The demands in equilibrium are given by

$$\widehat{d}_i^4 = \frac{-(64\lambda^6+48\lambda^5\mu-3000\lambda^3\mu^2+45000\lambda\mu^3-16875\mu^3(4\mu-25)+750\lambda^2\mu^2(6\mu-25)-8\lambda^4\mu(9\mu+275))}{(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

$$\widehat{d}_j^4 = \frac{(104\lambda^6-195\lambda^5\mu+6000\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)-750\lambda^2\mu^2(12\mu-115)+4\lambda^4\mu(72\mu-1375)}{(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

$$\widehat{d}_h^4 = \frac{(104\lambda^6-195\lambda^5\mu+6000\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)-750\lambda^2\mu^2(12\mu-115)+4\lambda^4\mu(72\mu-1375)}{(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

Their optimal investments in network capacities are

$$\begin{aligned}\widehat{s}_i^4 &= \frac{256\lambda^7-576\lambda^6\mu+101250\mu^4(4\mu-25)+144\lambda^4\mu^2(25+6\mu)-5400\lambda^2\mu^3(7\mu-50)}{15(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))} \\ &\quad + \frac{-33750\lambda\mu^3(12\mu-25)-32\lambda^5\mu(125+27\mu)+600\lambda^3\mu^2(63\mu-100)}{15(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}\end{aligned}$$

$$\widehat{s}_j^4 = \frac{-2(\lambda-3\mu)(52\lambda^6-96\lambda^5\mu+4200\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)+8\lambda^4\mu(18\mu-425)-75\lambda^2\mu^2(84\mu-925))}{15(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

$$\widehat{s}_h^4 = \frac{-2(\lambda-3\mu)(52\lambda^6-96\lambda^5\mu+4200\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)+8\lambda^4\mu(18\mu-425)-75\lambda^2\mu^2(84\mu-925))}{15(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

Their optimal investments in link capacities are

$$\widehat{s_i^j}^4 = \frac{-\lambda(64\lambda^6+48\lambda^5\mu-3000\lambda^3\mu^2+45000\lambda\mu^3-16875\mu^3(4\mu-25)+750\lambda^2\mu^2(6\mu-25)-8\lambda^4\mu(9\mu+275))}{5(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

$$\widehat{s_i^h}^4 = \frac{-\lambda(64\lambda^6+48\lambda^5\mu-3000\lambda^3\mu^2+45000\lambda\mu^3-16875\mu^3(4\mu-25)+750\lambda^2\mu^2(6\mu-25)-8\lambda^4\mu(9\mu+275))}{5(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

$$\widehat{s_j^i}^4 = \frac{\lambda(104\lambda^6-195\lambda^5\mu+6000\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)-750\lambda^2\mu^2(12\mu-115)+4\lambda^4\mu(72\mu-1375)}{5(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

$$\widehat{s_h^i}^4 = \frac{\lambda(104\lambda^6-195\lambda^5\mu+6000\lambda^3\mu^2-45000\lambda\mu^3+16875\mu^3(4\mu-25)-750\lambda^2\mu^2(12\mu-115)+4\lambda^4\mu(72\mu-1375)}{5(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))}$$

The reduced profits are given by

$$\begin{aligned}\widehat{\pi}_i^4 &= \left(\frac{-(32\lambda^4+23\lambda^3\mu-600\lambda\mu^2+225\mu^2(4\mu-25)+4\lambda^2\mu(9\mu-25))^2}{(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))^2}\right) \\ &\quad \left(\frac{272\lambda^6-768\lambda^5\mu+28800\lambda^3\mu^2-270000\lambda\mu^3+50625\mu^3(8\mu-75)-3600\lambda^2\mu^2(12\mu-125)+12\lambda^4\mu(96\mu-1525)}{450}\right)\end{aligned}$$

$$\widehat{\pi}_j^4 = \left(\frac{-(52\lambda^4-96\lambda^3\mu+1800\lambda\mu^2-675\mu^2(4\mu-25)+12\lambda^2\mu(12\mu-175)^2}{(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))^2}\right)$$
$$\left(\frac{80\lambda^6-48\lambda^5\mu+2400\lambda^3\mu^2-30000\lambda\mu^3+5625\mu^3(8\mu-75)+4\lambda^4\mu(18\mu-1225)-50\lambda^2\mu^2(72\mu-1265)}{450}\right)$$

$$\widehat{\pi}_h^4 = \left(\frac{-(52\lambda^4-96\lambda^3\mu+1800\lambda\mu^2-675\mu^2(4\mu-25)+12\lambda^2\mu(12\mu-175)^2}{(144\lambda^6-432\lambda^5\mu+15000\lambda^3\mu^2-135000\lambda\mu^3-11250\lambda^2\mu^2(2\mu-17)+50625\mu^3(4\mu-25)+8\lambda^4\mu(81\mu-1100))^2}\right)$$
$$\left(\frac{80\lambda^6-48\lambda^5\mu+2400\lambda^3\mu^2-30000\lambda\mu^3+5625\mu^3(8\mu-75)+4\lambda^4\mu(18\mu-1225)-50\lambda^2\mu^2(72\mu-1265)}{450}\right)$$

Appendix D

Parameter constraints

We require three restrictions on the equilibrium values.

(a) All the values of the variables at the equilibrium must be positive.

(b) There must be some connection failure for each IAP.

(c) There must be some congestion on each link as well as the NAP.

We have already imposed the constraints, $0 < \lambda < \mu \leq 1$ and $k \geq 0$. However for (a), (b) and (c) to hold, the aforesaid constraints are not sufficient. In fact, a sufficient condition is that the values of λ, μ, k must lie in the area defined by Figure 4.

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